

## Hairpin branes and D-branes behind the horizon

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Kazumi Okuyama<sup>a</sup> and Moshe Rozali<sup>ab</sup>

<sup>a</sup>*Department of Physics and Astronomy, University of British Columbia  
Vancouver, BC, V6T 1Z1, Canada*

<sup>b</sup>*Perimeter Institute for Theoretical Physics  
Waterloo, Ontario, N2L 2Y5, Canada*

*E-mail:* [kazumi@phas.ubc.ca](mailto:kazumi@phas.ubc.ca), [rozali@phas.ubc.ca](mailto:rozali@phas.ubc.ca)

**ABSTRACT:** We study Lorentzian D-particles in linear dilaton and the two dimensional black hole backgrounds. The D-particle trajectory follows an accelerated trajectory which is smeared by stringy corrections. For the black hole background we find that the portion of the trajectory behind the horizon appears to an asymptotic observer as ghost D-particle. This suggests a way of constructing a matrix model for the Lorentzian black hole background.

**KEYWORDS:** Conformal Field Models in String Theory, Black Holes, D-branes.

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## 1. Introduction

Time dependence, and more generally Lorentzian physics is an interesting and important subject in string theory. Generally calculations in string perturbation theory are performed in euclidean space, resulting in ambiguities once analytic continuation to Lorentzian signature space is performed. These ambiguities are physically important, encoding for example such issues as vacuum choice and Hawking radiation from black holes. It is interesting therefore to investigate this issue with an eye on the differences between string theory and ordinary quantum field theory in curved spacetime.

In this context, the hairpin branes, the topic of this paper, are interesting objects since they are time dependent and have an exact (tree level) worldsheet definition. The hairpin branes are defined in the linear dilaton backgrounds and their exact boundary states are known explicitly [1]. At the classical level, the hairpin brane is described by the boundary condition that the open string is attached to a fixed locus, which describes an hairpin shape. This picture is modified by  $\alpha'$  corrections that can be treated exactly. As noted in [2], the classical hairpin shape is smeared due to the presence of open string tachyon condensate near the tip of euclidean hairpin. In a recent paper [3], this picture

is made more precise by showing that the boundary CFT of hairpin brane is T-dual to a system with Liouville-like boundary potential. This can be thought of as a boundary analogue of the cigar/sine-Liouville duality. Interestingly, as we discuss in the text, even after including the worldsheet effects the exact boundary state has an expression as a superposition of classical trajectories with some smearing factor. This interplay between the classical trajectory and the exact boundary state is one of the main observations in this paper.

Upon Wick rotation to the Lorentzian signature, the hairpin brane becomes an accelerating brane, which moves towards the strong coupling end of the linear dilaton direction [4–6, 2]. Since it is a simple and clean time-dependent system, we can address many interesting questions related to the time-dependence in string theory. In particular, we consider the closed string field sourced by the brane and the closed string emission from the Lorentzian hairpin brane (which we find to be finite). We find that the classical Rindler horizon that exists for any accelerated trajectory is modified due to the stringy smearing of the D-brane trajectory.

There also exists an analogue of hairpin brane in two dimensional black hole background. As in the hairpin brane case, the exact boundary state on the euclidean black hole is known explicitly [7]. Again, the exact disk one-point function has a simple relation to the classical trajectory of brane [8]. However, in contrast to the hairpin brane case, the Wick rotation to the Lorentzian signature is a non-trivial problem due to the presence of horizons in the Lorentzian black hole. The euclidean section of black hole geometry only covers a causal patch outside the horizon in the Lorentzian geometry, thus the extension of the boundary state beyond the horizon is not unique. In this paper, we propose a prescription how to extend the boundary state to the entire Lorentzian black hole geometry, which amounts to imposing "transparency" when the D-particle crosses the horizon. Our method relies on the relation between the classical trajectory and the exact boundary state. Surprisingly, we find that the D-brane behind the horizon is a "ghost D-brane" introduced recently in [9].

This paper is organized as follows. In section 2, we consider hairpin branes in flat euclidean space and their Wick rotation to the Lorentzian space. We find an interesting interplay between the classical trajectory and the exact boundary state. Then we study the closed string emission from the Lorentzian hairpin brane and find that the emission rate is UV finite as in case of the rolling tachyon in the linear dilaton background.

In section 3, we consider D0-branes in 2d black hole background. Using the relation of classical trajectory and exact boundary state, we extend the definition of boundary state from one causal patch to the entire black hole geometry. In particular, we find that the D0-brane behind the horizon is a ghost D-brane. We also consider the closed string emission from D0-branes and discuss its dependence on the choice of closed string vacuum.

Section 4 is devoted to the discussion of possible significance of the ghost D-branes behind the horizon. Based on our result we speculate a possible matrix model dual of 2d Lorentzian black hole involving a supergroup.

## 2. Hairpin brane

In this section we discuss the hairpin brane in flat space with a linear dilaton background. We start by discussion of the features of the euclidean brane, followed by a discussion of Lorentzian features such as the closed string field and the (open and closed) string radiation emitted from the brane.

### 2.1 Euclidean hairpin

Let us first consider the euclidean version of hairpin brane. This is a boundary state in a system of two bosons  $X, Y$  (and possibly some additional compact CFT as needed to provide critical string background). The system has the linear dilaton background in the  $X$  direction

$$\Phi = -\frac{Q}{2}X. \quad (2.1)$$

The energy momentum tensor is

$$T = -\frac{1}{2}(\partial X)^2 - \frac{Q}{2}\partial^2 X - \frac{1}{2}(\partial Y)^2 \quad (2.2)$$

and the central charge is  $c = 2 + 3Q^2$ . The conformal weight of the tachyon vertex operator  $e^{-\frac{Q}{2}X + i\mathbf{p}\cdot\mathbf{X}}$  is  $\Delta = \frac{Q^2}{8} + \frac{1}{2}\mathbf{p}^2$ . Note also that the tachyons with positive and negative momenta  $p_x$  are unrelated, unlike the situation in Liouville theory where they are related by the reflection coefficient.

The hairpin brane is defined by the boundary condition

$$e^{-\frac{Q}{2}X} = C \cos \frac{Y}{2a} \quad (2.3)$$

where  $C$  is a constant characterizing the trajectory, and  $a$  is given by

$$a = \sqrt{1 + \frac{1}{Q^2}} \quad (2.4)$$

We see that in the weak coupling region  $X \sim +\infty$ ,  $Y$  approaches  $Y = \pm\pi a$ , hence the name "hairpin" brane. The coordinate  $Y$  is non-compact, but the trajectory extends only in the interval  $|Y| \leq \pi a$ , thus there is no periodicity in the euclidean time direction.

The boundary state for the hairpin brane is

$$\langle B_C(\mathbf{a}) | = \int d^2p \Psi_C(\mathbf{p}, \mathbf{a}) \langle\langle \mathbf{p} | \quad (2.5)$$

where  $\langle\langle \mathbf{p} |$  is the Ishibashi state built on the primary with momentum  $\mathbf{p}$ . The modulus  $\mathbf{a}$  of the boundary state characterizes the hairpin's tip location in spacetime (and should not be confused with the constant  $a$ ). We will for the most part choose coordinates such that  $\mathbf{a} = \mathbf{0}$ , which chooses a specific value for the constant  $C$  in the boundary condition (2.3).

The coefficients  $\Psi_C(\mathbf{p}, \mathbf{a})$  summarize the one point functions for the closed strings sourced by the brane. They are given by

$$\Psi_C(\mathbf{p}, \mathbf{a}) = e^{i\mathbf{p}\cdot\mathbf{a}} \frac{\frac{1}{Q}\Gamma(-2i\frac{p_x}{Q})\Gamma(1 - ip_x Q)}{\Gamma(\frac{1}{2} - i\frac{p_x}{Q} - ap_y)\Gamma(\frac{1}{2} - i\frac{p_x}{Q} + ap_y)} \quad (2.6)$$

where we omit an irrelevant overall constant for simplicity. In the semiclassical limit  $Q \rightarrow 0$  the Fourier transform of the wavefunction  $\Psi_C$  localizes on the trajectory (2.3). The factor  $\Gamma(-2i\frac{p_x}{Q})$  is interpreted as the effect of the bending of the brane (2.3) represented by the screening charge  $e^{-\frac{Q}{2}X}$  [2]. On the other hand, the factor  $\Gamma(1 - ip_x Q)$  represents the effect of the open string tachyon. Indeed, it is shown that the locations of the poles of  $\Gamma(1 - ip_x Q)$  are reproduced by the insertion of the screening operator [3]

$$S_B = \int_{\partial\Sigma} d\tau e^{-\frac{X}{Q}} (\sigma_+ e^{ia\tilde{Y}} + \sigma_- e^{-ia\tilde{Y}}) \quad (2.7)$$

where  $\sigma_{\pm}$  are Pauli matrices.

An interesting observable is the annulus amplitude which can be calculated in the closed string channel

$$\begin{aligned} Z(\mathbf{a}|\mathbf{a}') &= \langle B_C(\mathbf{a}) | q_c^{\frac{1}{2}(L_0 + \tilde{L}_0) - \frac{c}{24}} | B_C(\mathbf{a}') \rangle \\ &= \frac{1}{4} \int d^2 p e^{i\mathbf{p} \cdot (\mathbf{a} - \mathbf{a}')} \frac{\cos(2\pi a p_y) + \cosh(\frac{2\pi p_x}{Q})}{\sinh(\pi p_x Q) \sinh(\frac{2\pi p_x}{Q})} \frac{1}{\eta(q_c)^2} q_c^{\frac{1}{2}\mathbf{p}^2} \end{aligned} \quad (2.8)$$

Here  $q_c = e^{-2\pi t_c}$  is the annulus modular parameter in the closed string channel, and the factor  $\frac{1}{\eta(q_c)^2} q_c^{\frac{1}{2}\mathbf{p}^2}$  is the character of the Ishibashi state with momentum  $\mathbf{p}$ . Note that the characters in the bosonic case are known explicitly, a fact we will find useful below.

One can perform a modular transformation to the open string channel, with  $q_o = e^{-2\pi/t_c}$  being the open string modular parameter. We make use of the formula

$$\int \frac{d^D x}{(2\pi)^D} e^{i\mathbf{p} \cdot \mathbf{x}} \frac{1}{\eta(q_o)^D} q_o^{\frac{1}{8\pi^2} \mathbf{x}^2} = \frac{1}{\eta(q_c)^D} q_c^{\frac{1}{2}\mathbf{p}^2}. \quad (2.9)$$

giving the open string expression

$$Z(\mathbf{a}|\mathbf{a}') = \frac{1}{4} \int \frac{d^2 x d^2 p}{(2\pi)^2} e^{i\mathbf{p} \cdot \mathbf{x}} \frac{\cos(2\pi a p_y) + \cosh(\frac{2\pi p_x}{Q})}{\sinh(\pi p_x Q) \sinh(\frac{2\pi p_x}{Q})} \frac{1}{\eta(q_o)^2} q_o^{\frac{1}{8\pi^2} (\mathbf{x} - \mathbf{a} + \mathbf{a}')^2} \quad (2.10)$$

It is natural to classify the different terms in the annulus amplitude as coming from open strings stretched between either the same side or different sides of the hairpin. Indeed, the  $p_y$  integral leads to terms proportional to the  $\delta$ -functions

$$\delta(y \pm 2\pi a) \quad \text{or} \quad \delta(y). \quad (2.11)$$

The first pair of  $\delta$ -functions correspond to the open string stretched between the opposite sides of the D-brane  $Y = \pm\pi a$ , while  $\delta(y)$  comes from the open strings on the same side. This suggests a definition of "half-hairpin" branes corresponding to one of the branches of the hairpin only. To that end we can define the wave functions

$$\Psi_{\leftarrow}(\mathbf{p}, \mathbf{a}) = e^{i\mathbf{p} \cdot \mathbf{a}} \frac{1}{\sqrt{2}Q} \Gamma\left(-2i\frac{p_x}{Q}\right) \Gamma(1 - ip_x Q) \exp\left(\frac{\pi p_x}{Q} + \pi i a p_y\right) \quad (2.12)$$

for the first "incoming" branch and

$$\Psi_{\Rightarrow}(\mathbf{p}, \mathbf{a}) = e^{i\mathbf{p}\cdot\mathbf{a}} \frac{1}{\sqrt{2}Q} \Gamma\left(-2i\frac{p_x}{Q}\right) \Gamma(1 - ip_x Q) \exp\left(\frac{\pi p_x}{Q} - \pi i a p_y\right) \quad (2.13)$$

for the second "outgoing" branch. With these definitions the terms proportional to  $\delta(y)$  come from

$$\langle B_{\Rightarrow}(\mathbf{a}) | q^{\frac{1}{2}(L_0 + \tilde{L}_0) - \frac{c}{24}} | B_{\Rightarrow}(\mathbf{a}') \rangle \quad \text{or} \quad \langle B_{\Leftarrow}(\mathbf{a}) | q^{\frac{1}{2}(L_0 + \tilde{L}_0) - \frac{c}{24}} | B_{\Leftarrow}(\mathbf{a}') \rangle \quad (2.14)$$

whereas the terms proportional to  $\delta(y \pm 2\pi a)$  come from

$$\langle B_{\Leftarrow}(\mathbf{a}) | q^{\frac{1}{2}(L_0 + \tilde{L}_0) - \frac{c}{24}} | B_{\Rightarrow}(\mathbf{a}') \rangle \quad \text{or} \quad \langle B_{\Rightarrow}(\mathbf{a}) | q^{\frac{1}{2}(L_0 + \tilde{L}_0) - \frac{c}{24}} | B_{\Leftarrow}(\mathbf{a}') \rangle \quad (2.15)$$

Note that the half branes  $|B_{\Leftarrow}\rangle, |B_{\Rightarrow}\rangle$  are localized at  $Y = \pm\pi a$ . When Wick rotated to the Lorentzian signature, the half branes are analogous to a purely decaying D-branes and its time reversal (half S-branes [10]). We note that these boundary states satisfy the Cardy conditions separately.

These boundary states can be also constructed using the T-dual picture. Indeed, the euclidean hairpin brane is T-dual to the system with boundary potential (2.7) [3]. Therefore the half branes we define correspond, as for the rolling tachyon case, to boundary interactions including only one of the exponentials. We limit ourselves in what follows to discussion of the time-symmetric brane only .

Returning to the partition function (2.10) in the open string variables, the  $p_x$  integral can be performed as in eq.(4.11) of [11]. When  $\mathbf{a} = \mathbf{a}' = 0$ , the amplitude turns out to be a sum of two terms

$$Z = \frac{1}{2}(I_0 + I_1), \quad (2.16)$$

$$I_0 = \int_{-\infty}^{\infty} ds \rho_0(s) \frac{q_o^{\frac{1}{2}s^2}}{\eta(q_o)^2}, \quad I_1 = \int_{-\infty}^{\infty} ds \rho_1(s) \frac{q_o^{\frac{1}{2}s^2 + \frac{1}{2}a^2}}{\eta(q_o)^2} \quad (2.17)$$

where  $\rho_0$  and  $\rho_1$  are given by

$$\begin{aligned} \rho_0(s) &= \int_{-\infty}^{\infty} dp_x \frac{\cos(2\pi p_x s) \cosh\left(\frac{2\pi p_x}{Q}\right)}{2 \sinh(\pi p_x Q) \sinh\left(\frac{2\pi p_x}{Q}\right)} \\ \rho_1(s) &= \int_{-\infty}^{\infty} dp_x \frac{\cos(2\pi p_x s)}{2 \sinh(\pi p_x Q) \sinh\left(\frac{2\pi p_x}{Q}\right)} \end{aligned} \quad (2.18)$$

$\rho_0(s)$  is interpreted as the spectral density of open strings connecting the same side of the hairpin, and  $\rho_1(s)$  is the spectral density of the open strings connecting opposite sides of the hairpin brane. Note that the open string spectrum in  $I_1$  has a gap  $\frac{1}{2}a^2$  coming from the tension of open string stretched between two sides of the hairpin.

## 2.2 Open string production

As mentioned above, the euclidean hairpin brane is T-dual to the system with boundary potential [3]

$$S_B = \int_{\partial\Sigma} d\tau e^{-\frac{x}{Q}} (\sigma_+ e^{ia\tilde{Y}} + \sigma_- e^{-ia\tilde{Y}}) \quad (2.19)$$

Following [12], we analyze the open string dynamics in the mini-superspace approximation, that is we quantize the open string massive modes accounting for the time dependence of the zero modes. Those massive modes are solutions of the equation

$$\begin{pmatrix} -\partial_{\tilde{y}}^2 - \partial_x^2 + m^2 & e^{-\frac{x}{Q} + ia\tilde{y}} \\ e^{-\frac{x}{Q} - ia\tilde{y}} & -\partial_{\tilde{y}}^2 - \partial_x^2 + m^2 \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = 0 \quad (2.20)$$

where  $m$  is the open string mass coming from the transverse momentum and the stringy oscillators.

Solutions to (2.20) are given by the modified Bessel functions  $I_\nu, K_\nu$ . Requiring that the wavefunctions are localized at the weak coupling end, only  $K_\nu$  gives a sensible answer. Therefore we find the solution

$$\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} e^{\frac{i}{2}a\tilde{y}} K_\nu(2Qe^{-\frac{x}{2Q}}) \\ e^{-\frac{i}{2}a\tilde{y}} K_\nu(2Qe^{-\frac{x}{2Q}}) \end{pmatrix}, \quad \nu = Q\sqrt{a^2 + 4m^2} \quad (2.21)$$

This is reminiscent of the wavefunction in the Liouville theory [13], in particular the solution is not oscillatory in Lorentzian time. We therefore find that there is no open string production in the mini-space approximation.

## 2.3 Lorentzian hairpin brane

The Lorentzian brane is naturally defined via the Wick rotation of the position space wavefunction [5]

$$\begin{aligned} \tilde{\Psi}(\mathbf{x}) &= \int \frac{d^2p}{(2\pi)^2} e^{-i\mathbf{p}\cdot\mathbf{x}} \Psi(\mathbf{p}) \\ &= \frac{1}{2\pi a Q^2} \left(2 \cos \frac{y}{2a}\right)^{-1-\frac{2}{Q^2}} \exp\left[-\frac{x}{Q} - e^{-\frac{x}{Q}} \left(2 \cos \frac{y}{2a}\right)^{-\frac{2}{Q^2}}\right] \\ &\xrightarrow{y=ix^0} \frac{1}{2\pi a Q^2} \left(2 \cosh \frac{x^0}{2a}\right)^{-1-\frac{2}{Q^2}} \exp\left[-\frac{x}{Q} - e^{-\frac{x}{Q}} \left(2 \cosh \frac{x^0}{2a}\right)^{-\frac{2}{Q^2}}\right]. \end{aligned} \quad (2.22)$$

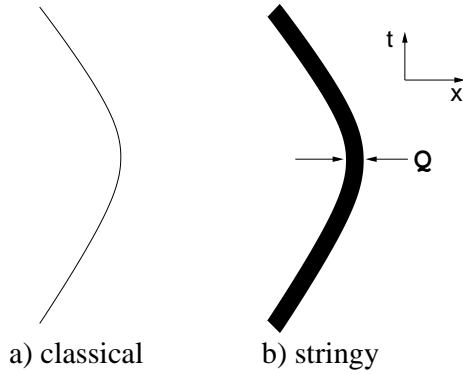
This is approximately localized on the Lorentzian trajectory

$$e^{-\frac{Q}{2}X} = 2 \cosh \frac{X^0}{2a} \quad (2.23)$$

where we chose coordinates to simplify the form of the trajectory.

By an inverse Fourier transform, the momentum space wavefunction for the Lorentzian hairpin brane is given by

$$\begin{aligned} \Psi(p, \omega) &= \int dx^0 dx e^{i\omega x^0 + ipx} \tilde{\Psi}(x^0, x) \\ &= \frac{1}{2\pi} \frac{\Gamma(1-ipQ)}{\Gamma\left(1+i\frac{2p}{Q}\right)} \Gamma\left(\frac{1}{2} + i\left(\frac{p}{Q} + a\omega\right)\right) \Gamma\left(\frac{1}{2} + i\left(\frac{p}{Q} - a\omega\right)\right) \end{aligned} \quad (2.24)$$



**Figure 1:** Stringy smearing of the D-brane trajectory. The classical trajectory  $e^{-\frac{Q}{2}x} = 2 \cosh \frac{t}{2a}$  is smeared by the weight function  $w(s)$  with width  $Q$ .

Interestingly, the wavefunction (2.22) can be written as the smearing of the trajectory (2.23) over its modulus (expressing the maximal location of the trajectory, or equivalently the initial velocity at some point in time). Indeed

$$\begin{aligned} \Psi(x, x^0) &= \frac{1}{4\pi a Q^2 \cosh \frac{x^0}{2a}} \int_{-\infty}^{\infty} ds w(s) \delta \left( x - s + \frac{2}{Q} \log \left( 2 \cosh \frac{x^0}{2a} \right) \right) \\ &= \frac{1}{4\pi a Q} \int_{-\infty}^{\infty} ds w(s) \delta \left( e^{-\frac{Q}{2}(x-s)} - 2 \cosh \frac{x^0}{2a} \right) \end{aligned} \quad (2.25)$$

where the smearing function  $w(s)$  is given by <sup>1</sup>

$$w(s) = \exp \left( -\frac{s}{Q} - e^{-\frac{s}{Q}} \right) \quad (2.26)$$

In other words, the wavefunction is a superposition of the trajectory  $e^{-\frac{Q}{2}(x-s)} = 2 \cosh \frac{x^0}{2a}$  with the weight  $w(s)$ . We can see that  $w(s)$  is localized around  $s = 0$

$$w(s) \sim \exp \left( -\frac{s^2}{2Q^2} \right) \quad \text{for } s \sim 0 \quad (2.27)$$

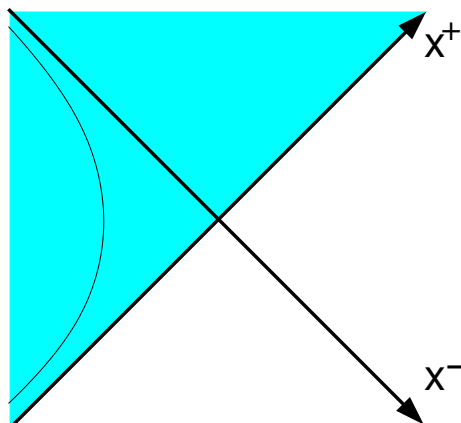
Note that the width of distribution is  $\Delta s = Q$  [2] and  $w(s)$  decays to zero as  $s \rightarrow \pm\infty$ . This smearing comes from the factor  $\Gamma(1 - ip_x Q)$  in the momentum space wavefunction (2.24), which in turn is identified in euclidean space as the effect of open string winding tachyon.

We note that if we interpret the wavefunction  $w(s) = \exp \left( -\frac{s}{Q} - e^{-\frac{s}{Q}} \right)$  as the profile of the zero mode  $s$ , the profile of the D-brane itself is given by<sup>2</sup>  $e^{-e^{-\frac{s}{Q}}}$ . This has the semiclassical form  $e^{-I(s)}$ , with  $I(s) = e^{-\frac{s}{Q}}$  being the action of the screening charge  $e^{-\frac{X}{Q}}$ .

<sup>1</sup>This can be shown using the expression  $\Gamma(z) = \int_{-\infty}^{\infty} dt \exp(-zt - e^{-t})$ .

<sup>2</sup>In a classical field theory the zero mode profile is a derivative of the classical solution.





**Figure 2:** The trajectory in the  $Q \rightarrow 0$  limit is contained in one quadrant  $x^+ < 0, x^- < 0$ . Therefore, the closed string radiation is observed only in the shaded region, i.e. the future lightcone of the trajectory. This picture is completely changed when including stringy effects.

## 2.4 The closed string field

As discussed in [14], the classical closed string field  $|C\rangle$  sourced by the boundary state  $|B\rangle$  is found by solving the equation  $(L_0 + \bar{L}_0)|C\rangle = |B\rangle$ , imposing retarded boundary conditions. As is familiar with discussions of accelerated trajectories, there is an effective Rindler horizon in spacetime - causality prevents any signal from reaching a part of the spacetime. In our case that part includes all of the weakly coupled asymptotic region.

To see that effect we concentrate on the lowest mode of the closed string field, namely the closed string tachyon. Neglecting for the moment the stringy smearing of the trajectory, the Lorentzian boundary state can be written in lightcone coordinates. Defining  $p_{\pm} = \frac{p}{Q} \pm a\omega$  we find in the  $Q \rightarrow 0$  limit

$$\Psi(p_+, p_-) = \frac{1}{2\pi} \frac{\Gamma(\frac{1}{2} + ip_+) \Gamma(\frac{1}{2} + ip_-)}{\Gamma(1 + ip_+ + ip_-)} \quad (2.28)$$

This localizes the position space wave function on the trajectory  $e^{x^+} + e^{x^-} = 1$ , where  $x^{\pm}$  are the positions conjugate to the lightcone momenta  $p_{\mp}$  respectively, namely  $x^{\pm} = \frac{Q}{2}x \mp \frac{x_0}{2a}$ . This is of course the trajectory (2.23) written in lightcone coordinates. Note that this implies that both  $x^{\pm}$  are negative, so that the trajectory is localized on one quadrant of the two dimensional Minkowski space, as shown in figure 2.

The retarded propagator for a two-dimensional massless field is

$$G_R(p_+, p_-) \sim \frac{1}{(p_- - i\epsilon)(p_+ + i\epsilon)} \quad (2.29)$$

where we omit an overall constant and use the fact that in the  $Q \rightarrow 0$  limit  $a \rightarrow \frac{1}{Q}$ . Therefore the massless closed string field generated by our trajectory is proportional to

$$T(x^+, x^-) \sim \int dp_- dp_+ \exp(-ip_- x^+ - ip_+ x^-) \frac{1}{(p_- - i\epsilon)(p_+ + i\epsilon)} \frac{\Gamma(\frac{1}{2} + ip_+) \Gamma(\frac{1}{2} + ip_-)}{\Gamma(1 + ip_+ + ip_-)} \quad (2.30)$$

The integral is easily evaluated, as are similar integrals for the massive closed string fields. We will not be interested in the detailed structure of the closed string field, rather just in its causal structure. To this end we look at the poles of the integrand of (2.30) and note that they appear only for  $\text{Im}(p_-) > 0$  (whereas there are poles in both upper and lower half planes of  $p_+$ , when taking into account the poles in the gamma functions). Therefore it is clear that the closed string field vanishes for  $x^- > 0$ . This fits with our expectations from causality, as drawn in figure 2.

The effect of stringy corrections to the boundary state smear the trajectory as described above, and therefore it is clear that the resulting closed string field will be everywhere non-vanishing. This can be seen by noticing that the additional factor in the boundary state  $\Gamma(1 - iQ^2(p_+ + p_-))$  has poles both in the upper and lower half planes of  $p_-$ . Indeed, an estimate of the profile of the closed string field at large positive  $x^-$ , based on the first pole in the lower half plane, gives  $T \sim e^{-\frac{2}{Q}x^-}$ .

Therefore we find that the closed string field has exponential tail in a region naively forbidden by causality, resulting from stringy smearing of the trajectory of the D-brane.

## 2.5 Closed string radiation

Next we wish to calculate the imaginary part of annulus amplitude between two hairpin branes. We start with the  $(X^0, X)$  part

$$Z^{X^0 X} = \int dp d\omega |\Psi(p, \omega)|^2 \eta(q_c)^{-2} q_c^{\frac{1}{2}(p^2 - \omega^2)} \quad (2.31)$$

Following [11] we rewrite this in terms of the characters in the open string channel. Using (2.9) we obtain

$$\begin{aligned} Z^{X^0 X} &= \int dk dE dp d\omega |\Psi(p, \omega)|^2 \cos(2\pi k p) \cos(2\pi E \omega) \eta(q_o)^{-2} q_o^{\frac{1}{2}(k^2 - E^2)} \\ &= \frac{1}{2aQ} \int dk dE \frac{\sinh \frac{2\pi E}{aQ^2}}{\sinh \frac{\pi E}{a} (\cosh \frac{2\pi k}{Q} + \cosh \frac{2\pi E}{aQ^2})} \eta(q_o)^{-2} q_o^{\frac{1}{2}(k^2 - E^2)} \end{aligned} \quad (2.32)$$

where  $q_o$  is the open string modulus, and  $E, k$  are respectively the energy and momentum in the open string channel (roughly the T-duals of  $\omega, p$ ).

As the integral over  $q_o$  is ill-defined, due to the non-unitary factor  $X^0$  in the CFT, we need to make sense of this expression. We do that by euclidean rotation of the variable  $E$ , resulting in

$$Z^{X^0 X} = \frac{1}{2aQ} \int dk dE \frac{\sin \frac{2\pi E}{aQ^2}}{\sin \frac{\pi E}{a} (\cosh \frac{2\pi k}{Q} + \cos \frac{2\pi E}{aQ^2})} \eta(q_o)^{-2} q_o^{\frac{1}{2}(k^2 + E^2)} \quad (2.33)$$

where  $E$  is now the euclidean frequency, and it is integrated over a Feynman contour, which includes the poles of the expression for positive values of  $E$ .

In order to obtain a critical string background we couple the  $(X^0, X)$  CFT to  $D$  free bosons (or more generally some compact CFT with central charge  $D$ ) such that

$$D + 2 + 3Q^2 = 26. \quad (2.34)$$

The contribution from  $D$  free boson and the  $bc$  ghosts is

$$\begin{aligned} Z^{X^i+bc} &= \int d^{D-p} k_{\perp} \eta(q_c)^{2-D} q_c^{\frac{1}{2}k_{\perp}^2} \\ &= t_o^{1-\frac{p}{2}} \eta(q_o)^{2-D}. \end{aligned} \quad (2.35)$$

Here we assumed that the  $p$  bosons satisfy the Neumann boundary condition and  $D-p$  bosons satisfy the Dirichlet condition. Putting it all together the total annulus amplitude is given by

$$\mathcal{Z} = \frac{1}{2aQ} \int_0^{\infty} \frac{dt_o}{t_o} t_o^{-\frac{p}{2}} \eta(q_o)^{-D} \int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} dE \frac{\sin \frac{2\pi E}{aQ^2}}{\sin \frac{\pi E}{a} \left( \cosh \frac{2\pi k}{Q} + \cos \frac{2\pi E}{aQ^2} \right)} q_o^{\frac{1}{2}(k^2+E^2)} \quad (2.36)$$

In the form (2.36) the partition function is well-defined and one can extract its imaginary part. Indeed, due to the poles at  $E = na$  ( $n \in \mathbb{Z}$ ) on the real  $E$ -axis, and using the Feynman contour,  $\mathcal{Z}$  acquires the imaginary part

$$\text{Im}\mathcal{Z} = \frac{1}{2Q} \int_0^{\infty} \frac{dt_o}{t_o} t_o^{-\frac{p}{2}} \eta(q_o)^{-D} \int_{-\infty}^{\infty} dk \sum_{n=1}^{\infty} \frac{(-1)^n \sin \frac{2\pi n}{Q^2}}{\cosh \frac{2\pi k}{Q} + \cos \frac{2\pi n}{Q^2}} q_o^{\frac{1}{2}(k^2+n^2a^2)} \quad (2.37)$$

The imaginary part is to be interpreted as the total radiation of closed strings. We now discuss some of its features.

First, it is interesting to check whether the total radiation is finite. To that end we look at the open string IR limit  $t_o \rightarrow \infty$ , the amplitude is then dominated by the  $n = 1$  pole, and behaves as

$$\text{Im}\mathcal{Z} = -\frac{1}{Q} \int_0^{\infty} \frac{dt_o}{t_o} t_o^{-\frac{p}{2}} \int_{-\infty}^{\infty} dk \frac{\sin \frac{2\pi}{Q^2}}{\cosh \frac{2\pi k}{Q} + \cos \frac{2\pi}{Q^2}} e^{-\pi t_o (k^2+a^2-\frac{D}{12})} + \dots \quad (2.38)$$

We note that the decay rate is exponentially suppressed with an exponent which is positive definite

$$a^2 - \frac{D}{12} = \left(1 + \frac{1}{Q^2}\right) - \frac{24 - 3Q^2}{12} = \left(\frac{1}{Q} - \frac{Q}{2}\right)^2 \quad (2.39)$$

This exponential suppression can be simply understood in the open string channel, since the ground state of the stretched open strings is massive, as in [2]. To see this, let us look at the action of open string tachyon at the weak coupling end

$$\mathcal{L} = -e^{\frac{Q}{2}x} \left[ (\partial_x T)^2 + m(x)^2 T^2 \right] \quad (2.40)$$

where  $m(x)^2$  is the position dependent mass

$$m(x)^2 = -\frac{1}{2} + \frac{1}{\pi^2} a^2 \arccos^2 \left( e^{-\frac{Qx}{2}} \right) \quad (2.41)$$

The first term is the usual tachyon mass and the second term is the tension of open string stretched between two sides of the euclidean hairpin brane. In terms of the canonically normalized field  $\tilde{T} = e^{\frac{Q}{4}x} T$ , the effective mass becomes

$$\tilde{m}(x)^2 = m(x)^2 + \frac{Q^2}{16} \quad (2.42)$$

The tachyon mass at the weak coupling side is

$$\lim_{x \rightarrow \infty} \tilde{m}(x) = -\frac{1}{2} + \frac{a^2}{4} + \frac{Q^2}{16} = \frac{1}{4} \left( \frac{1}{Q} - \frac{Q}{2} \right)^2 \quad (2.43)$$

This agrees with (2.39) up to a normalization of momentum.

We conclude then that the closed string radiation is finite, and becomes infinite exactly at the point where the Lorentzian hairpin brane becomes non-normalizable ( $Q = \sqrt{2}$ ). This is reminiscent of the discussion of the rolling tachyon in [11], and contradicts the results in [5].<sup>3</sup>

As the decay rate is finite we expect it to be dominated by radiation to few of the lowest mass levels of the closed string. As the characters are exactly known we could calculate the emission rate of any closed string field. For example, in order to extract the contribution of the light closed string modes, we take the closed string IR limit  $t_o \rightarrow 0$ , the amplitude behaves as

$$\begin{aligned} \text{Im}\mathcal{Z} &= \frac{1}{2Q} \int_0^{t_o} \frac{dt_o}{t_o} t_o^{\frac{D-p}{2}} \left( e^{\frac{2\pi D}{24t_o}} + D e^{\frac{2\pi(D-24)}{24t_o}} + \dots \right) \\ &\times \int_{-\infty}^{\infty} dk \sum_{n \in \mathbb{Z}} \frac{(-1)^n \sin \frac{2\pi n}{Q^2}}{\cosh \frac{2\pi k}{Q} + \cos \frac{2\pi n}{Q^2}} q_o^{\frac{1}{2}(k^2 + n^2 a^2)} \end{aligned} \quad (2.44)$$

The first term is the closed string tachyon contribution. The second term is exponentially suppressed because  $D - 24 = -3Q^2 < 0$ . This is a manifestation of the fact that the graviton in the linear dilaton background is screened and is effectively massive.

Finally, we discuss the details of the radiation as function of time or frequency. To this end we can calculate the moments of the radiation, or equivalently the generating function for those moments. That generating function is obtained by inserting a factor  $e^{i\omega t}$  in (2.33). Repeating the steps above gives an expression of the form (2.36) with the replacement

$$\frac{1}{\sin \frac{\pi E}{a}} \rightarrow f(t, E) = \frac{2 \sin \frac{\pi E}{a} \cosh \frac{t}{2a}}{\cosh \frac{t}{a} - \cos \frac{2\pi E}{a}} \quad (2.45)$$

The Fourier transform of  $f(t, E)$  is

$$\tilde{f}(\omega, E) = \int dt e^{-i\omega t} f(t, E) = 2\pi a \frac{e^{-2\pi\omega E}}{1 + e^{2\pi a\omega}} \quad (2.46)$$

Therefore the annulus amplitude is written as

$$\mathcal{Z}(t) = \int d\omega e^{i\omega t} F(\omega) \quad (2.47)$$

$$F(\omega) = \frac{1}{2aQ} \int_0^{\infty} \frac{dt_o}{t_o} t_o^{-\frac{p}{2}} \eta(q_o)^{-D} \int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} dE \frac{\tilde{f}(\omega, E) \sin \frac{2\pi E}{aQ^2}}{\cosh \frac{2\pi k}{Q} + \cos \frac{2\pi E}{aQ^2}} q_o^{\frac{1}{2}(k^2 + E^2)} \quad (2.48)$$

---

<sup>3</sup>In addition to discussing a slightly different model, the analysis in [5] uses saddle point approximation which becomes invalid at large mass levels.

We note that the expression (2.46) is a Boltzmann factor written in terms of the open string (euclidean) frequency. This suggests that the system, when formulated in the open string variables, is thermal with temperature

$$T = \frac{1}{2\pi a} \tag{2.49}$$

which is the expected Unruh temperature of the accelerated brane. As another piece of evidence we note that even though the system has no euclidean time periodicity in the original variables, it does have such periodicity in the dual variables (2.19). We note that this temperature is lower than the closed string Hagedorn temperature for the physical range of the parameter  $Q$ .

### 3. D-branes on the two dimensional black hole

In this section we discuss the hairpin brane in the two-dimensional black hole background. In addition to discussing the features of the brane outside the horizon, previously also discussed in [8], we suggest an analytic continuation of the boundary state emphasizing the geometrical trajectory of the particle. With this analytic continuation we find a surprising result: the portion of the trajectory inside the horizon looks to an asymptotic observer as a ghost D-brane [9].

#### 3.1 D1-Branes on the euclidean Cigar

The euclidean cigar  $SL(2, \mathbb{R})/U(1)$  CFT is described by the background metric and dilaton

$$ds^2 = k(d\rho^2 + \tanh^2 \rho d\theta^2), \quad \Phi = -\log \cosh \rho \tag{3.1}$$

and the central charge is

$$c = 2 + 6b^2, \quad b^2 = \frac{1}{k-2} \tag{3.2}$$

D-branes in the background were constructed in [7]. We are interested in the D1-brane on the cigar whose disk one-point function of primary operator  $\phi_n^p$  is given by

$$\Psi_{(\rho_0, \theta_0)}(p, n) = \frac{\Gamma(ip)\Gamma(1+ipb^2)}{\Gamma(\frac{1}{2} + i\frac{p}{2} + \frac{n}{2})\Gamma(\frac{1}{2} + i\frac{p}{2} - \frac{n}{2})} e^{in\theta_0} (e^{-ip\rho_0} + (-1)^n e^{ip\rho_0}) \tag{3.3}$$

where  $\rho_0 (> 0)$  and  $\theta_0$  are constant parameters specifying the classical shape of D1-brane

$$\cos(\theta - \theta_0) \sinh \rho = \sinh \rho_0 \tag{3.4}$$

Following [8], we rewrite (3.3) as the integral of mini-superspace closed string wavefunctions over the trajectories. In the mini-superspace approximation, the eigenfunction of the target space Laplacian  $\Delta = \frac{1}{e^{-2\Phi}\sqrt{g}}\partial_i(e^{-2\Phi}\sqrt{g}g^{ij}\partial_j)$  is given by

$$\phi_n^p = \phi_{L,n}^p + \mathcal{R}_0(p, n)\phi_{R,n}^p \tag{3.5}$$

where  $\phi_{L,n}^p$  and  $\phi_{R,n}^p$  are the left and right moving modes

$$\begin{aligned}\phi_{L,n}^p &= e^{in\theta} (\sinh \rho)^{-1-ip} F\left(\frac{1+ip+n}{2}, \frac{1+ip-n}{2}, 1+ip, -\frac{1}{\sinh^2 \rho}\right) \\ \phi_{R,n}^p &= e^{in\theta} (\sinh \rho)^{-1+ip} F\left(\frac{1-ip-n}{2}, \frac{1-ip+n}{2}, 1-ip, -\frac{1}{\sinh^2 \rho}\right)\end{aligned}\quad (3.6)$$

and  $\mathcal{R}_0(p, n)$  in (3.5) is the mini-superspace reflection amplitude

$$\mathcal{R}_0(p, n) = \frac{\Gamma(ip)\Gamma(\frac{1-ip+n}{2})^2}{\Gamma(-ip)\Gamma(\frac{1+ip+n}{2})^2}\quad (3.7)$$

which related spacetime left and right movers (as the coordinate  $\rho$  is semi-infinite). Note that  $\phi_n^p$  and  $\phi_n^{-p}$  are related by the reflection

$$\phi_n^p = \mathcal{R}_0(p, n)\phi_n^{-p}.\quad (3.8)$$

Therefore, the boundary state is expanded by the Ishibashi states  $\langle\langle\phi_n^p|$  with  $p > 0$

$$\langle B| = \int_0^\infty dp \sum_{n \in \mathbb{Z}} \Psi_{(\rho_0, \theta_0)}(p, n) \langle\langle\phi_n^p|\quad (3.9)$$

In [8], it was shown that the disk one-point function for the D1-brane (3.3) is written as the integral of mini-superspace wavefunction  $\phi_n^p(\rho, \theta)$  over the classical trajectory (3.4)

$$\Psi_{(\rho_0, \theta_0)}(p, n) = \Gamma(1+ipb^2) \int d\mu \delta(\cos(\theta - \theta_0) \sinh \rho - \sinh \rho_0) \phi_n^p(\rho, \theta)\quad (3.10)$$

where the measure  $d\mu$  is given by

$$\int d\mu = \int e^{-2\Phi} \sqrt{g} d\rho d\theta = \int_0^\infty \sinh \rho d(\sinh \rho) \int_0^{2\pi} d\theta\quad (3.11)$$

This is analogous to Fourier transform of the boundary state in the hairpin case. As in the hairpin case, the factor  $\Gamma(1+ipb^2)$  in (3.10) has the effect of smearing the trajectory and it can be attributed to the condensate of open string tachyon. It is curious that in both cases the sole effect of  $\alpha'$  corrections is summarized in rigid smearing of the classical trajectories.

We can compute the annulus amplitude between two D1-branes as in the hairpin case:

$$\begin{aligned}Z(\rho_0, \theta_0 | \rho'_0, \theta'_0) &= \int_0^\infty dp \sum_{n \in \mathbb{Z}} \frac{q_c^{\frac{b^2 p^2 + n^2}{4}}}{\eta(q_c)^2} \Psi_{(\rho_0, \theta_0)}(p, n) \Psi_{(\rho'_0, \theta'_0)}^*(p, n) \\ &= \int ds \frac{q_o^{\frac{b^2 s^2}{4}}}{\eta(q_o)^2} \left[ d_0(s | \rho_0, \rho'_0) \sum_{w \in \mathbb{Z}} q_o^{k(w + \frac{\theta_0 - \theta'_0}{2\pi})^2} + d_1(s | \rho_0, \rho'_0) \sum_{w \in \mathbb{Z} + \frac{1}{2}} q_o^{k(w + \frac{\theta_0 - \theta'_0}{2\pi})^2} \right]\end{aligned}\quad (3.12)$$

where  $d_0$  and  $d_1$  are the open string density of states defined by

$$\begin{aligned} d_0(s|\rho_0, \rho'_0) &= \int_0^\infty dp \cos(\pi ps) \frac{\cosh(\frac{\pi p}{2}) \cos(p\rho_0) \cos(p\rho'_0)}{\sinh(\frac{\pi p}{2}) \sinh(\pi b^2 p)} \\ d_1(s|\rho_0, \rho'_0) &= \int_0^\infty dp \cos(\pi ps) \frac{\sinh(\frac{\pi p}{2}) \sin(p\rho_0) \sin(p\rho'_0)}{\cosh(\frac{\pi p}{2}) \sinh(\pi b^2 p)}. \end{aligned} \quad (3.13)$$

We can interpret the  $d_0$  term in (3.12) as the open string stretched between the same side of D1-brane, while  $d_1$  term is the contribution of the open string stretched between the opposite sides of D1-brane. As noted in [1], the structure of the annulus amplitude for the D1-branes on the cigar is very similar to the annulus amplitude of the euclidean hairpin brane.

### 3.2 The lorentzian black hole

The two-dimensional Lorentzian black hole is obtained from the euclidean one (3.1) by a Wick rotation  $\theta = it$  [15]

$$ds^2 = k(d\rho^2 - \tanh^2 \rho dt^2), \quad \Phi = -\log \cosh \rho. \quad (3.14)$$

In this Schwarzschild coordinate system, the trajectory of D0-brane can be obtained by the Wick rotation of the euclidean D1-brane (3.4)

$$\cosh(t - t_0) \sinh \rho = \sinh \rho_0 \quad (3.15)$$

However, in order to discuss the global properties of the geometry, and the full trajectory of the D-brane, it is useful to introduce the Kruskal coordinate

$$u = \sinh \rho e^t, \quad v = \sinh \rho e^{-t}. \quad (3.16)$$

In terms of these coordinates, the black hole geometry is given by

$$ds^2 = 2k \frac{dudv}{1+uv}, \quad e^{2\Phi} = \frac{1}{1+uv} \quad (3.17)$$

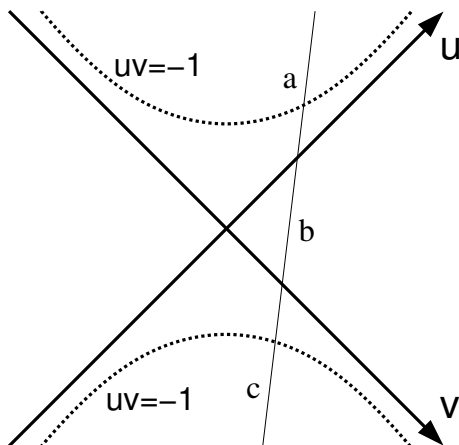
The  $uv$ -plane is divided into various regions due to the presence of the horizon at  $uv = 0$  (see figure 3.). In particular,

$$\begin{aligned} \text{outside the horizon : } & uv > 0 \\ \text{behind the horizon : } & uv < 0. \end{aligned} \quad (3.18)$$

There is a spacelike singularity at  $uv = -1$ . Note that in string theory it may be sensible to include the region beyond the singularity  $uv < -1$ . In particular we will find that the boundary state of the D-particle does not see the singularity at this level of approximation.

The closed string wavefunctions in the Kruskal coordinate are given by the hypergeometric functions

$$\begin{aligned} R_\omega^p &= u^{-\nu_-} v^{-\nu_+} F\left(\nu_-, \nu_+, \nu_- + \nu_+; -\frac{1}{uv}\right) \\ L_\omega^p &= u^{-\bar{\nu}_+} v^{-\bar{\nu}_-} F\left(\bar{\nu}_-, \bar{\nu}_+, \bar{\nu}_- + \bar{\nu}_+; -\frac{1}{uv}\right) \end{aligned} \quad (3.19)$$



**Figure 3:** The two-dimensional black hole in the Kruskal coordinate. The horizon is at  $uv = 0$  and the singularity is at  $uv = -1$ . The trajectory of D0-brane in this coordinate is a straight line, which consists of three parts ( $a$ ,  $b$  and  $c$  in the figure). Correspondingly, the disk one-point function has two parts,  $\Psi_{\text{outside}} = b$ ,  $\Psi_{\text{behind}} = a + c$ . The classical trajectory is smeared by the open string tachyon.

Here we defined

$$\nu_{\pm} = \frac{1}{2} - i \frac{p \pm \omega}{2} \tag{3.20}$$

As discussed in [16], these eigenfunctions (3.19) can be analytically continued to the entire  $uv$ -plane. This basis of functions satisfy natural boundary conditions at infinity, namely they represent purely left or right movers at large radial distances. In contrast to the euclidean case, mode functions with  $p > 0$  and  $p < 0$  are independent. Since  $L_{\omega}^p$  and  $R_{\omega}^p$  are related by the sign flip of momentum

$$L_{\omega}^{-p} = R_{\omega}^p \tag{3.21}$$

we can take the basis of Ishibashi states as

$$\left\{ |L_{\omega}^p\rangle\rangle, |R_{\omega}^p\rangle\rangle \right\}_{p>0, \omega \in \mathbb{R}} = \left\{ |L_{\omega}^p\rangle\rangle \right\}_{p, \omega \in \mathbb{R}}. \tag{3.22}$$

Note that the reflection relation (3.8) satisfied by the euclidean modes amounts to imposing Hartle-Hawking boundary conditions at the outer horizon,<sup>4</sup> which is a natural set of boundary condition needed when discussing the physics outside the horizon. As one of our goal is to understand the D-particle in the full geometry, we do not impose any such reflection relations and leave the modes independent.

Another natural set of eigenfunctions  $U_{\omega}^p, V_{\omega}^p$  is characterized by boundary conditions at the horizon, they are purely incoming or outgoing modes at the horizon. This basis is related to  $L_{\omega}^p, R_{\omega}^p$  by

$$\begin{aligned} U_{\omega}^p &= L_{\omega}^p + \mathcal{R}_0(p, \omega) R_{\omega}^p \\ V_{\omega}^p &= R_{\omega}^p + \mathcal{R}_0^*(p, \omega) L_{\omega}^p \end{aligned} \tag{3.23}$$

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<sup>4</sup>It is interesting that those boundary conditions receive stringy corrections.



where  $\mathcal{R}_0$  is the mini-superspace reflection amplitude

$$\mathcal{R}(p, \omega) = -\frac{B(\nu_+, \nu_-)}{B(\bar{\nu}_+, \bar{\nu}_-)} S(p, \omega), \quad S(p, \omega) = \frac{\cosh(\pi \frac{p-\omega}{2})}{\cosh(\pi \frac{p+\omega}{2})} \quad (3.24)$$

and  $B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$  denotes the Euler beta-function.

We note that these wavefunctions all diverge logarithmically near the singularity at  $uv = -1$ . We will find that this divergence disappears when the closed string modes are combined in the wave packet corresponding to the wavefunction of the D-particle.

### 3.3 D-particles and ghost particles in the lorentzian black hole

The D0-brane trajectory (3.15) becomes, when expressed in Kruskal coordinates, a straight line on the  $uv$ -plane [17, 18]

$$ue^{-t_0} + ve^{t_0} = 2 \sinh \rho_0. \quad (3.25)$$

This can be understood by the fact that the effective metric seen by the brane is flat:  $ds_{D0}^2 = dudv$ , the conformal factor of the metric and the dilaton precisely cancel each other. In particular, before including stringy or quantum corrections there is no special status to the singularity at  $uv = -1$ .

In order to analytically continue the euclidean boundary state (3.10), we need to define the disk one-point function of  $L_\omega^p$  for the D0-brane in the black hole background. Since there are more such modes in the Lorentzian black hole background than the euclidean section (as modes with positive and negative momenta are unrelated), there is some freedom in determining these one point function.

Our prescription relies on the geometric interpretation of the trajectory: the one point functions are obtained by integrating the mini-superspace eigenfunction  $L_\omega^p(u, v)$  along the classical trajectory. This in effect imposing "transparent horizon" boundary conditions, in other words the D-brane propagation through the horizon is smooth:

$$\Psi^L(p, \omega) = \Gamma(1 + ipb^2) \int d\mu \delta\left(ue^{t_0} + ve^{-t_0} - 2 \sinh \rho_0\right) L_\omega^p(u, v) \quad (3.26)$$

where the measure  $d\mu$  is

$$\int d\mu = \int e^{-2\Phi} \sqrt{-g} dudv = \int dudv \quad (3.27)$$

Note that the measure in the  $uv$ -coordinate is flat, thus there is no singularity in the measure factor. This is related to the cancelation between the dilaton and conformal factor in the action for the D-particle.

In (3.26) we have to specify the integration region of  $u, v$  variables. There are two natural choices of integration region: we can integrate over the whole  $uv$ -plane, or just over the region outside the horizon  $uv > 0$ . In the former case the eigenfunction  $L_\omega^p(u, v)$  should be extended to the entire  $uv$ -plane as in [16]. Explicitly, we extend  $L_\omega^p(u, v)$  from

one causal patch  $u, v > 0$  to the entire  $uv$ -plane as follows:

$$L_{\omega}^p(u, v) = \begin{cases} e^{-\pi i \bar{v}_+} (-u)^{-\bar{v}_+} v^{-\bar{v}_-} F(\bar{v}_+, \bar{v}_-, \bar{v}_+ + \bar{v}_-; -1/uv) & u < 0, v > 0 \\ e^{\pi i \bar{v}_-} u^{-\bar{v}_+} (-v)^{-\bar{v}_-} F(\bar{v}_+, \bar{v}_-, \bar{v}_+ + \bar{v}_-; -1/uv) & u > 0, v < 0 \\ e^{\pi i (\bar{v}_- - \bar{v}_+)} (-u)^{-\bar{v}_+} (-v)^{-\bar{v}_-} F(\bar{v}_+, \bar{v}_-, \bar{v}_+ + \bar{v}_-; -1/uv) & u < 0, v < 0 \end{cases} \quad (3.28)$$

Our phase convention is such that the Schwarzschild time  $t$  is shifted by  $-\pi i$  when crossing a horizon. The extension of  $R_{\omega}^p$  is obtained similarly by requiring (3.21). Note that this is the prescription that maps naturally to real time thermal field theory.

Let us now consider the contribution to the boundary state from the regions outside the horizon and behind the horizon separately

$$\begin{aligned} \langle B | \equiv \langle B |_{\text{outside}} &= \int_{-\infty}^{\infty} \frac{dp}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \langle \langle L_{\omega}^p | \Psi^L(p, \omega) \rangle_{\text{outside}} \\ \langle B |_{\text{behind}} &= \int_{-\infty}^{\infty} \frac{dp}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \langle \langle L_{\omega}^p | \Psi^L(p, \omega) \rangle_{\text{behind}} \end{aligned} \quad (3.29)$$

where the disk one-point functions of  $L_{\omega}^p$  are given by

$$\begin{aligned} \Psi^L(p, \omega)_{\text{outside}} &= \Gamma(1 + ipb^2) \int_{uv>0} dudv \delta(ue^{t_0} + ve^{-t_0} - 2 \sinh \rho_0) L_{\omega}^p(u, v) \\ \Psi^L(p, \omega)_{\text{behind}} &= \Gamma(1 + ipb^2) \int_{uv<0} dudv \delta(ue^{t_0} + ve^{-t_0} - 2 \sinh \rho_0) L_{\omega}^p(u, v) \end{aligned} \quad (3.30)$$

Using the explicit form of the eigenfunctions (3.19), we find a remarkable property<sup>5</sup>

$$\Psi^L(p, \omega)_{\text{behind}} = -\Psi^L(p, \omega)_{\text{outside}} \quad (3.31)$$

It should be emphasized that the integral behind the horizon is finite even though it includes the contribution near and behind the singularity. The finiteness of the contribution from singularity can be seen as follows: while the function  $L_{\omega}^p$  blows up as  $\log(1 + uv)$  near  $uv = -1$ , its integral behaves like  $(1 + uv) \log(1 + uv)$  which is finite near  $uv = -1$ . This is consistent with the classical picture that the D0-brane does not see the singularity. As the stringy corrections amount to smearing over the classical trajectory, we find that inclusion of these  $\alpha'$  corrections does not change the picture.

The striking relation (3.31) suggests that the portion of the D-brane behind the horizon (and the singularity) can be interpreted by an asymptotic observer as a “ghost D-brane” introduced in [9]. Namely, the D-brane behind the horizon is the same as the D-brane outside the horizon with an overall minus sign

$$|B\rangle_{\text{behind}} = -|B\rangle \quad (3.32)$$

Note that in the  $c = 1$  matrix model context such ghost D-brane is identified as the hole state [19, 20]. Therefore, we propose to identify the D-brane outside the horizon as a

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<sup>5</sup>This conclusion does not change if we choose  $t \rightarrow t + \pi i$  instead of  $t \rightarrow t - \pi i$  for the crossing of horizon.

particle and the D-brane behind the horizon as a hole. This suggests a relation to the black hole complementarity [21]. It is also reminiscent of picturing the Hawking radiation as pair (particle-hole) creation near the horizon.

We note also that once the D-particle crosses the horizon it becomes visible to the second asymptotic region of the geometry, which is related to the first one by complex time shift and time reversal [22]. Excitations of this boundary appear ghostlike precisely as they appear in real time thermal field theory [23, 24].

Since the dilaton diverges near the singularity we do expect quantum corrections to reintroduce some singular behavior. We expect therefore corrections to the exact relations (3.31), but those corrections will be small for suitable wavepackets. In addition, the fact that the segment inside the horizon is ghostlike, is not going to be modified, even in the absence of the relation (3.31).

The relation (3.31) imposes, via the Cardy condition, unusual quantization condition on the open strings stretched between the portions of the D-brane outside and inside the horizon. Indeed the relative sign means that the ground state of such open string that crosses the horizon is fermionic. It would be interesting to understand the open string sector on the D-brane in more detail.

Returning to the full boundary state (3.26), the explicit form of the disk one-point function of  $L_\omega^p$  is found to be

$$\Psi^L(p, \omega)_{\text{outside}} = e^{-i\omega t_0 - ip\rho_0} B(\nu_+, \nu_-) \Gamma(1 + ipb^2) \quad (3.33)$$

This agrees with the result in [8] obtained from a different method of analytic continuation. Indeed, the portion of our D0-brane outside the horizon corresponds to the time-symmetric brane in [8].

As was the case for the hairpin brane in flat space, the factor  $\Gamma(1 + ipb^2)$  can be interpreted similarly as the smearing of the trajectory. To see this, note that (3.33) is rewritten as <sup>6</sup>

$$\Psi^L(p, \omega)_{\text{outside}} = \int_{-\infty}^{\infty} ds w(s) e^{-i\omega t_0 - ip(\rho_0 + s)} B(\nu_+, \nu_-) \quad (3.34)$$

where the weight  $w(s)$  has the same form as the hairpin case (2.26)

$$w(s) = \frac{1}{b^2} \exp\left(-\frac{s}{b^2} - e^{-\frac{s}{b^2}}\right) = \frac{d}{ds} \exp\left(-e^{-\frac{s}{b^2}}\right) \quad (3.35)$$

In other words, the parameter  $\rho_0$  in the classical trajectory fluctuates with width  $\Delta\rho_0 = b^2$ .

Note that in our definition the boundary state  $|B\rangle$  is independent of the choice of basis for the closed string modes. In particular, it is independent of the choice of closed string vacuum. This is consistent with the fact that the boundary state can be seen as

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<sup>6</sup>One is tempted to write the one-point function in the form

$$\Psi^L(p, \omega)_{\text{outside}} = \int_{-\infty}^{\infty} ds w(s) \int_{uv>0} dudv \delta\left(ue^{-t_0} + ve^{t_0} - 2 \sinh(\rho_0 + s)\right) L_\omega^p.$$

However, this is not correct because the  $uv$ -integral is proportional to  $e^{-ip|\rho_0+s|}$ , which is not analytic in  $s$ .

the external source of all closed string fields, which is clearly independent of the vacuum choice. Therefore, we can expand  $|B\rangle$  in any basis. For example, for later use we now expand  $\langle B|$  in terms of the basis  $U_\omega^p, V_\omega^p$ .

The mini-superspace relation (3.23) is modified in the exact treatment of CFT

$$\begin{aligned} |U_\omega^p\rangle &= |L_\omega^p\rangle + \mathcal{R}(p, \omega)|R_\omega^p\rangle \\ |V_\omega^p\rangle &= |R_\omega^p\rangle + \mathcal{R}^*(p, \omega)|L_\omega^p\rangle \end{aligned} \quad (3.36)$$

where  $\mathcal{R}(p, \omega)$  is the exact reflection coefficient

$$\mathcal{R}(p, \omega) = \mathcal{R}_0(p, \omega) \frac{\Gamma(1 + ipb^2)}{\Gamma(1 - ipb^2)} \quad (3.37)$$

From (3.36) we can easily find the disk one-point function of  $U_\omega^p, V_\omega^p$

$$\begin{aligned} \langle B|U_\omega^p\rangle &= \Psi^L(p, \omega) + \mathcal{R}(p, \omega)\Psi^L(-p, \omega) \\ \langle B|V_\omega^p\rangle &= \Psi^L(-p, \omega) + \mathcal{R}^*(p, \omega)\Psi^L(p, \omega) \end{aligned} \quad (3.38)$$

Therefore in terms of the basis (3.36) the boundary state can be expressed as

$$\begin{aligned} \langle B| &= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_0^{\infty} \frac{dp}{2\pi} \frac{1}{1 - |\mathcal{R}(p, \omega)|^2} \left[ \langle U_\omega^p | \left( \Psi^L(p, \omega) - \mathcal{R}^*(p, \omega)\Psi^L(-p, \omega) \right) \right. \\ &\quad \left. \langle V_\omega^p | \left( \Psi^L(-p, \omega) - \mathcal{R}(p, \omega)\Psi^L(p, \omega) \right) \right] \end{aligned} \quad (3.39)$$

### 3.4 Annulus amplitude and radiation

Let us consider the annulus amplitude between two D0-branes outside the horizon. After coupling the black hole system with  $D$  free bosons in such a way that

$$D + 2 + 6b^2 = 26, \quad (3.40)$$

the imaginary part of the annulus amplitude, calculated in the open string channel as before, turns out to be essentially the same as the hairpin brane case:

$$\begin{aligned} \mathcal{Z} &= \langle B | \frac{1}{L_0 + \tilde{L}_0 + i\epsilon} | B \rangle \\ &= \int_0^{\infty} \frac{dt_o}{t_o} t_o^{-\frac{p}{2}} \eta(q_o)^{-D} \int ds dE q_o^{\frac{E^2}{4k} + \frac{b^2 s^2}{4}} \frac{\sin \frac{\pi E}{kb^2}}{\sin \frac{\pi E}{k} (\cosh(\pi s) + \cos \frac{\pi E}{kb^2})} \\ \longrightarrow \text{Im} \mathcal{Z} &= k \int_0^{\infty} \frac{dt_o}{t_o} t_o^{-\frac{p}{2}} \eta(q_o)^{-D} \int ds \sum_{n \in \mathbb{Z}} q_o^{\frac{kn^2}{4} + \frac{b^2 s^2}{4}} \frac{(-1)^n \sin \frac{\pi n}{b^2}}{\cosh(\pi s) + \cos \frac{\pi n}{b^2}} \end{aligned} \quad (3.41)$$

In particular this imaginary part is UV finite since

$$\tilde{m}_\infty^2 = \frac{k}{4} - \frac{D}{24} = \frac{1}{4} \left( \frac{1}{b} - b \right)^2 \geq 0. \quad (3.42)$$

However, in computing the closed string radiation we have to be careful about the vacuum choice which exists here due to the presence of the horizon. This is most easily seen in the closed string channel: when choosing a particular vacuum, the vacuum independent source term  $|B\rangle$  is decomposed into positive and negative frequency parts

$$|B\rangle = |B^{(+)}\rangle + |B^{(-)}\rangle. \tag{3.43}$$

With the Feynman prescription of the closed string propagator  $(L_0 + \tilde{L}_0 + i\epsilon)^{-1}$ , each component  $|B^{(\pm)}\rangle$  contributes to the amplitude with a definite sign. To see the effect of vacuum choice simply, let us consider the particle limit of the amplitude (3.41), evaluated in the closed string channel

$$\mathcal{Z} \sim \int d^2x d^2y \langle B|x\rangle G(x,y) \langle y|B\rangle \tag{3.44}$$

Here we approximate the boundary state by the mini-superspace wavefunction

$$\langle B|x\rangle = \int \frac{dpd\omega}{(2\pi)^2} \Psi^L(p,\omega) L_\omega^p(x) \tag{3.45}$$

We also set  $D = 0, k = \frac{9}{4}$  for simplicity. Namely, we consider the purely two dimensional black hole.

As discussed in [16], the Green function  $G(x,y)$  depends on the vacuum. For example, in the Hartle-Hawking vacuum  $G(x,y)$  reads (for  $t_x > t_y$ )

$$G_H(x,y) = \int_{-\infty}^{\infty} dp \rho(p) \left[ (1 + N_\omega) U_p(x) U_p(y)^* + N_\omega U_p^*(x) U_p(y) \right] \tag{3.46}$$

where

$$U_p = U_{\omega=-3p>0}^p, \quad U_p = V_{\omega=3p>0}^p \tag{3.47}$$

and  $N_\omega$  is the Boltzmann distribution function

$$N_\omega = \frac{1}{e^{2\pi\omega} - 1} \tag{3.48}$$

The measure  $\rho(p)$  is given by

$$\rho(p) = \frac{\pi}{6} |\tanh 2\pi p| \tag{3.49}$$

In this Hartle-Hawking vacuum the closed strings are in thermal equilibrium with the black hole, with temperature  $T_H$ . As the temperature of the hairpin brane is lower than that, the system is not at equilibrium and the radiation as seen by an asymptotic observer is not thermal, as can be easily seen from the explicit form of the one point functions.

On the other hand, in the Schwarzschild vacuum there is no asymptotic closed string radiation and  $G(x,y)$  has no thermal factor  $N_\omega$

$$G_S(x,y) = \int_{-\infty}^{\infty} dp \rho(p) U_p(x) U_p(y)^* \tag{3.50}$$

hence the radiation seen by this observer comes entirely from the D-particle and it is thermal (in the open string variables, as explained in the previous section).

Therefore we find that the radiation as seen by the Schwarzschild observer is precisely what we obtain from the calculation in the open string channel, and is very similar to the case of the hairpin brane in flat space. Apparently the calculation in the open string channel involves implicitly a vacuum choice for the closed string sector. It is interesting to investigate whether the open string calculation can yield more general such vacua.

#### 4. Discussion and outlook

There is no evidence of black hole formation in the high energy scattering of closed string tachyons in the singlet sector of  $c = 1$  matrix model [25–27]. Therefore, it is widely believed that the 2d black hole is related to the non-singlet sector of matrix model. In [28], based on the FZZ duality between the cigar and the sine-Liouville, it is argued that the euclidean 2d black hole is described by a matrix model with Wilson loops added to the action. However, a direct Lorentzian description of 2d black hole is still lacking. Our result of D0-branes in Lorentzian black hole may shed light on this problem. From the general philosophy of open-closed duality, the Lorentzian black hole should be dual to the theory on a large number of D0-branes on the black hole background. Our result suggests that the Lorentzian black hole is dual to a system of D0-branes and ghost D0-branes, i.e.  $U(n|m)$  supermatrix quantum mechanics.<sup>7</sup> In particular, the open string stretched between D0-branes outside the horizon and D0-branes behind the horizon is fermionic. Note that the appearance of fermionic open string in bosonic string theory is not new. Indeed it is known that the open string between FZZT-brane and ZZ-brane in bosonic minimal string theory is fermionic [29]. This is also consistent with the proposal that the effect of Wilson loops in the KKK matrix model is reproduced by introducing non-singlet fermions into a Lorentzian matrix model [30, 31]. We leave the study of supermatrix model and its relation to 2d black hole as an interesting future problem.

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<sup>7</sup>Note that in [17, 26] it is proposed that the black hole state in the  $c = 1$  matrix model is represented by a large number of soft particle-hole pairs.

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